



Experimental review on moment analyses

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Moments of the photon energy spectrum in $B \rightarrow X_s \gamma$ decays, of the hadronic mass spectrum and of the lepton energy spectrum in $B \rightarrow X_c \ell \nu$ decays are sensitive to the masses of the heavy quarks as well as to the non-perturbative parameters of the heavy quark expansion. Several measurements have been performed both at the $\Upsilon(4S)$ resonance and at Z^0 center of mass energies. They provide constraints on the non-perturbative parameters, give a test of the consistency of the theoretical predictions and of the underlying assumptions and allow to reduce the uncertainties in the extraction of $|V_{cb}|$.

1 Introduction

The Operation Product Expansion is a powerful tool to study the dynamics of heavy flavour hadrons and represents a basis for extracting the $|V_{cb}|$ element of the CKM mixing matrix from inclusive semileptonic B decays. In this framework inclusive observables are expressed in terms of quark masses and non-perturbative effects are described by expectation values of heavy quark operators.

As an example, when a double expansion in α_s and $1/m_B$ is used, the non-perturbative parameter introduced at first order is $\bar{\Lambda}$, which is related to the energy of the light degrees of freedom inside the heavy meson. At second order λ_1 and λ_2 are introduced, which are related to the kinetic energy of the b quark inside the meson and to the chromo-magnetic coupling of the b spin to the light degrees of freedom, respectively. At third order other six parameters appear: $\rho_1, \rho_2, \mathcal{T}_1, \mathcal{T}_2, \mathcal{T}_3$ and \mathcal{T}_4 . Only λ_2 is relatively well known, from the $B^* - B$ meson mass difference.

In the determination of $|V_{cb}|$ from the inclusive semileptonic decay width

$$\Gamma(b \rightarrow c \ell \nu) = |V_{cb}|^2 \gamma_{th} = BR(b \rightarrow c \ell \nu) / \tau_b$$

the uncertainties on the values of the non-perturbative parameters and on the assumptions underlying the theoretical prediction, notably quark-hadron duality, provided until recently the dominant error contribution [1]. The current experimental uncertainties on the measurements of the semileptonic branching ratio and of the B lifetime give a contribution of only about 1%.

However other inclusive variables like the moments of the photon energy spectrum in $B \rightarrow X_s \gamma$ decays and moments of the hadronic mass spectrum and of the

lepton energy spectrum in $B \rightarrow X_c \ell \nu$ decays are sensitive to the masses of the heavy quarks and to the non-perturbative parameters of the heavy quark expansion. Measurements of spectral moments give constraints on the non-perturbative parameters and allow to reduce the uncertainties in the extraction of $|V_{cb}|$ [2]. Moreover, the comparison of results obtained from different measurements provides a test of the consistency of the theoretical predictions and of the underlying assumptions.

The first measurements of spectral moments have been performed at the $\Upsilon(4S)$ by the CLEO Collaboration. New preliminary results have been presented in summer 2002 by BABAR and DELPHI. Moments of second and third order have been measured, having sensitivities to several parameters of the heavy quark expansion. For experimental reasons, measurements performed up to now at the $\Upsilon(4S)$ require a minimum value of the lepton energy of about 1.5 GeV and thus have to compare to theoretical calculations which are also restricted to a truncated lepton spectrum.

2 CLEO

The photon energy spectrum in $B \rightarrow X_s \gamma$ decays has been studied with $9.1 fb^{-1}$ of data collected at the $\Upsilon(4S)$ resonance [3] with the CLEO detector. The large background of high energy photons from continuum processes has been suppressed by using several techniques and the remaining part has been estimated with Monte Carlo and subtracted. The photon spectrum is shown Fig. 1. After correcting for the experimental resolution, efficiency and smearing due to the B momentum, the first and second moment, for $E_\gamma > 2$ GeV, have been measured to be:

$$\begin{aligned} \langle E_\gamma \rangle &= (3.346 \pm 0.032 \pm 0.011) \text{ GeV} \\ \langle (E_\gamma - \langle E_\gamma \rangle)^2 \rangle &= (0.0226 \pm 0.0066 \pm 0.0020) \text{ GeV}^2 \end{aligned}$$

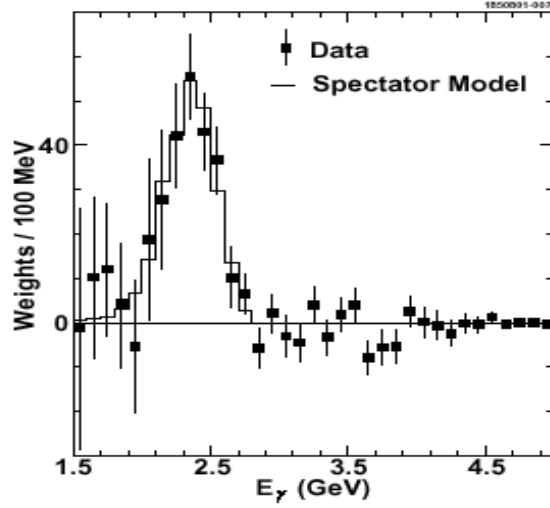


Figure 1. CLEO. Photon energy spectrum in $B \rightarrow X_s \gamma$ decays, in the laboratory frame, after background subtraction [3].

The hadronic mass spectrum has been reconstructed in $B \rightarrow X_c \ell \nu$ decays using 3.2 fb^{-1} of data collected at the $\Upsilon(4S)$ resonance [4]. The hermeticity of the CLEO detector has been exploited in reconstructing the neutrino by using momentum conservation of the entire event. Once the B momentum, which is small ($\sim 300 \text{ MeV}/c$), is set to zero, the mass of the hadronic system is determined from the lepton and neutrino momentum vectors alone as: $\tilde{M}_X^2 = M_B^2 + M_{\ell\nu}^2 - 2E_B E_{\ell\nu}$. Fig. 2 shows the \tilde{M}_X^2 distribution, for background-corrected data. With the

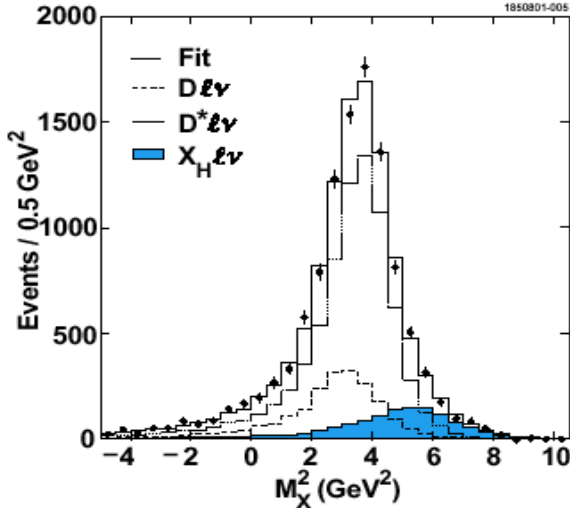


Figure 2. CLEO. Measured hadronic mass distribution for background corrected data. Different contributions in the Monte Carlo are also shown [4].

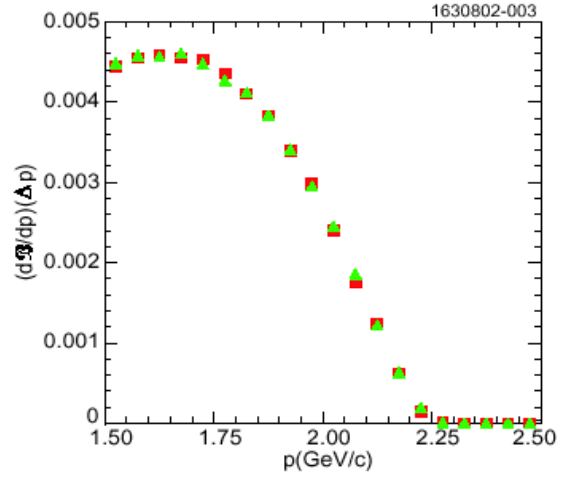


Figure 3. CLEO. Corrected electron (triangles) and muon (squares) energy spectra, in the B rest frame [7].

requirement $p_\ell > 1.5 \text{ GeV}/c$ the first two moments of the hadronic mass spectrum have been measured to be:

$$\begin{aligned} \langle M_X^2 - m_D^2 \rangle &= (0.251 \pm 0.023 \pm 0.062) \text{ GeV}^2 \\ \langle (M_X^2 - m_D^2)^2 \rangle &= (0.576 \pm 0.048 \pm 0.163) \text{ GeV}^4 \end{aligned}$$

where $m_{\bar{D}}$ is the spin-averaged D meson mass $m_{\bar{D}} = (M_D + 3M_D^*)/4$.

Using for the first moments the expressions given in refs. [5] and [6], which are up to $1/m_B^3$ and $\alpha_s^2 \beta_0$ order, the following constraints on $\bar{\Lambda}$ and λ_1 have been derived:

$$\begin{aligned} \bar{\Lambda} &= (0.35 \pm 0.07 \pm 0.10) \text{ GeV} \\ \lambda_1 &= (-0.236 \pm 0.071 \pm 0.078) \text{ GeV}^2 \end{aligned}$$

where the first uncertainty is the experimental one on the two moments and the second is from the theoretical expressions.

New results on leptons have been recently published in ref. [7]. The momentum spectra of electron and muon have been analysed using 3.1 fb^{-1} of data collected at the $\Upsilon(4S)$ resonance. A minimum momentum of $p_\ell > 1.5 \text{ GeV}/c$ has been required, in order to ensure good efficiency and to reduce the contamination due to secondary leptons. In Fig. 3 the electron and muon spectra are shown, after background subtraction.

The measurement of two moments of truncated lepton spectra, R_0 and R_1 , have been used to constrain $\bar{\Lambda}$ and λ_1 [8]

$$\begin{aligned} R_0 &= 0.6187 \pm 0.0014 \pm 0.0016 \\ R_1 &= (1.7810 \pm 0.0007 \pm 0.0009) \text{ GeV}. \end{aligned}$$

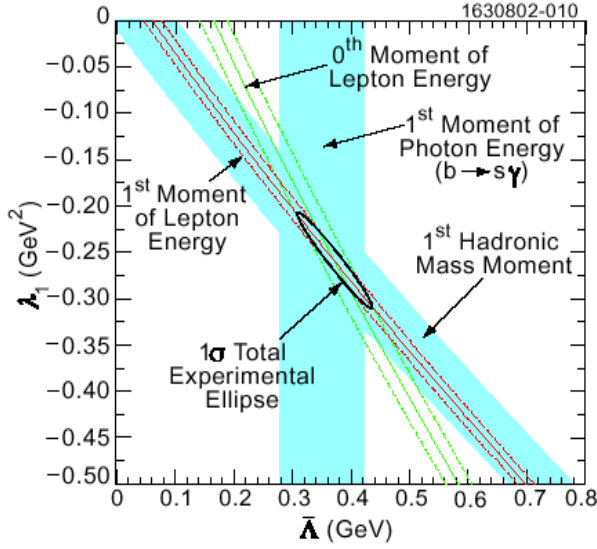


Figure 4. Constraints on the $\bar{\Lambda}$ and λ_1 parameters obtained from CLEO measurements. The $\Delta\chi^2 = 1$ contour for the total experimental uncertainty is shown for the measurements of R_0 and R_1 moments. The bands corresponding to the measurements of the first moment of photon energy spectrum and hadronic mass spectrum, respectively, are also shown.

where

$$R_0 = \frac{\int_{1.5\text{GeV}}^{1.7\text{GeV}} d\Gamma_{sl}/dE_\ell}{\int_{1.5\text{GeV}} d\Gamma_{sl}/dE_\ell} \quad R_1 = \frac{\int_{1.5\text{GeV}}^{1.7\text{GeV}} E_\ell d\Gamma_{sl}/dE_\ell}{\int_{1.5\text{GeV}} d\Gamma_{sl}/dE_\ell}.$$

The dependence of R_0 and R_1 on the parameters $\bar{\Lambda}$ and λ_1 is shown in Fig. 4. The allowed region corresponds to the values:

$$\begin{aligned} \bar{\Lambda} &= (0.39 \pm 0.03 \pm 0.06 \pm 0.12) \text{ GeV} \\ \lambda_1 &= (-0.25 \pm 0.02 \pm 0.05 \pm 0.14) \text{ GeV}^2 \end{aligned}$$

where the quoted uncertainties are in order statistical, systematic and theoretical. The derived constraints are in good agreement with those determined from the first moments of photon energy spectrum and of hadronic mass spectrum measurements. Using as input $\Gamma_{sl}^{exp} = (0.43 \pm 0.01) \times 10^{-10} \text{ MeV}$ the following value of $|V_{cb}|$ has been obtained:

$$|V_{cb}| = (40.8 \pm 0.5 \pm 0.4 \pm 0.9) \times 10^{-3}$$

where the first uncertainty is experimental, the second is from the uncertainties in the non-perturbative parameters $\bar{\Lambda}$ and λ_1 and the third is the theoretical uncertainty determined by varying the input parameters within their respective errors. The uncertainty related to the truncation of the perturbative series is not included.

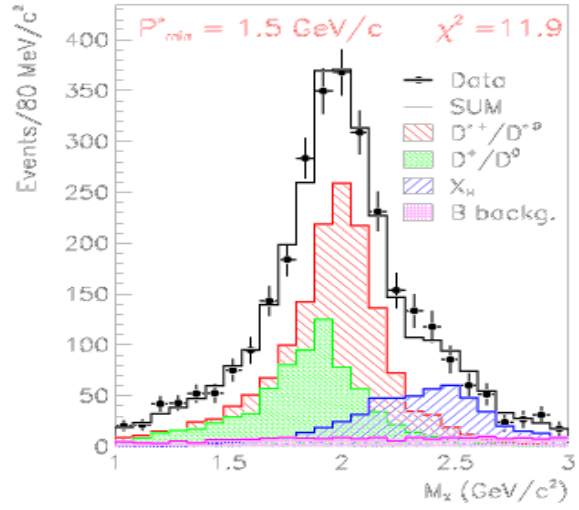


Figure 5. BABAR. Hadronic mass distribution for $p_\ell > 1.5 \text{ GeV}/c$.

3 BABAR

A preliminary measurement of the spectrum of the hadronic mass in $B \rightarrow X_c \ell \nu$ decays has been presented by the BABAR Collaboration, based on 51 fb^{-1} of data collected at the $\Upsilon(4S)$ resonance [9]. Profiting of the high statistics, one of the B mesons (B_{reco}) is fully reconstructed in a hadronic channel and the semileptonic decay of the other B is studied. The mass of the hadronic system is reconstructed by summing all tracks in the event except the lepton and B_{reco} and the mass resolution is improved by performing a 2-C kinematic fit to the whole event that imposes four-momentum conservation. The mass distribution is shown in Fig. 5. The mass distribution is fitted to the sum of four contributions accounting for decays in D^* and D mesons, in higher mass states X_H , including both resonant D^{**} and non resonant $D^*\pi$ states, and background. Monte Carlo simulation is used to derive the shape of the signal and background contributions. A minimum momentum has been required for the lepton, which has been varied in the range 0.9 to $1.5 \text{ GeV}/c$. The contributions measured from the fit (f_{D^*} , f_D and f_{X_H}) are used to determine the first moment of hadronic mass spectrum as:

$$f_{D^*}(m_{D^*}^2 - m_D^2) + f_D(m_D^2 - m_D^2) + f_{X_H}(M_{X_H}^2 - m_D^2)$$

With the result obtained for $p_\ell > 1.5 \text{ GeV}/c$, and using as a constraint on $\bar{\Lambda}$ the one derived by CLEO from $B \rightarrow X_s \gamma$ spectra $\bar{\Lambda} = (0.35 \pm 0.13) \text{ GeV}$, λ_1 has been determined to be:

$$\lambda_1 = (-0.17 \pm 0.06 \pm 0.07) \text{ GeV}^2$$

in good agreement with the λ_1 value derived by CLEO. However the dependence of the first hadronic mass moment on p_ℓ^{min} has been found to be quite steeper than what is predicted by theory [6] and a fit to the data at different p_ℓ^{min} with λ_1 and Λ as free parameters gives results incompatible with the previous ones. A new analysis is under-way on BABAR data which will give more insight into this question.

4 DELPHI

The first measurement of moments in b hadron semileptonic decays at Z^0 center of mass energies has been performed by the DELPHI Collaboration. The main advantage at the Z^0 pole is the large boost acquired by the b quark ($E_B \sim 30$ GeV) which gives access in the laboratory frame to the low region of the lepton energy spectrum. This makes these results both easier to interpret and complementary to those obtained at the $\Upsilon(4S)$. The challenge in this case is the complete reconstruction of the B system.

Moments of lepton (electrons and muons) energy spectrum have been measured in inclusive b -hadron semileptonic decays [10]. Secondary vertices have been reconstructed using an iterative procedure. The B energy has been determined adding to the energy at the charm vertex the lepton energy and the neutrino energy, evaluated from the event missing energy. The B direction has been estimated from both the reconstructed B momentum and the B decay flight direction and leptons have been boosted in the B rest frame. After unfolding the resolution smearing, the first, second and third moments have been determined to be:

$$\begin{aligned}\langle E_\ell \rangle &= (1.383 \pm 0.012 \pm 0.008) \text{ GeV} \\ \langle (E_\ell - \langle E_\ell \rangle)^2 \rangle &= (0.192 \pm 0.005 \pm 0.010) \text{ GeV}^2 \\ \langle (E_\ell - \langle E_\ell \rangle)^3 \rangle &= (-0.029 \pm 0.005 \pm 0.005) \text{ GeV}^3\end{aligned}$$

For the measurement of hadronic mass moments $B_d^0 \rightarrow D^{**}\ell\bar{\nu}$ events have been studied [11]. D^{**} events have been reconstructed in the three channels: $D^0\pi^+$, $D^+\pi^-$, $D^{*+}\pi^-$ with the D^0 , D^+ and D^{*+} meson decays fully reconstructed. Leptons have been required to have a momentum greater than 2 GeV/c in the laboratory frame. The separation of the signal from the background has been achieved by means of a discriminant variable based on the topological properties of the secondary vertex.

A fit to the variable $\Delta M = m(D^{(*)}\pi) - m(D^{(*)})$ has been performed, with contributions of narrow and broad resonant states D_0^{*+} , D_1^{*+} , D_1^+ and D_2^+ as well as non resonant $D\pi$ states free in the fit. Constraints on available measurements on narrow states have also been applied. The total rate for D^{**} production resulting from the fit

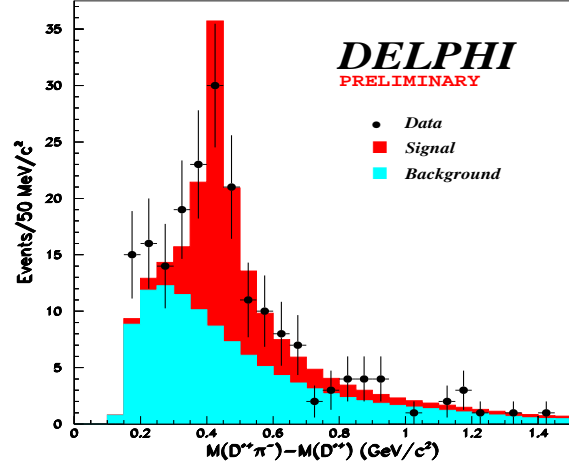


Figure 6. DELPHI. ΔM distributions in the $D^{*+}\pi^-$ reconstructed decay channel.

$$BR(B_d^0 \rightarrow D^{**}\ell\bar{\nu}) = (2.6 \pm 0.5 \pm 0.6)\%$$

is well compatible with previous measurements. The ΔM distribution in one of the reconstructed decay channels is shown in Fig. 6. From the fitted mass distributions the first three moments of the D^{**} mass distribution have been evaluated. The relation:

$$\langle m_X^n \rangle = p_D m_D^n + p_{D^*} m_{D^*}^n + p_{D^{**}} \langle m_{D^{**}}^n \rangle$$

has been used to add the $b \rightarrow D\ell\bar{\nu}$ and the $b \rightarrow D^*\ell\bar{\nu}$ contributions, where p_D and p_{D^*} are the relative branching fractions derived from published results and $p_{D^{**}}$ is obtained by imposing the constraint $p_D + p_{D^*} + p_{D^{**}} = 1$ and using the above measurement.

The following preliminary results have been obtained:

$$\begin{aligned}\langle m_X^2 - m_D^2 \rangle &= (0.534 \pm 0.041 \pm 0.074) \text{ GeV} \\ \langle (m_X^2 - \langle m_X^2 \rangle)^2 \rangle &= (1.23 \pm 0.16 \pm 0.15) \text{ GeV}^2 \\ \langle (m_X^2 - \langle m_X^2 \rangle)^3 \rangle &= (2.97 \pm 0.67 \pm 0.48) \text{ GeV}^3\end{aligned}$$

where the first uncertainty is statistic and the second is systematic.

From the measured spectral moments constraints on the non-perturbative parameters of the OPE have been obtained [12]. The formalism based on low scale running masses [13] which does not rely on a $1/m_c$ expansion has been used. Here $m_b(\mu)$ and $m_c(\mu)$ are independent parameters, μ_π^2 has a similar meaning than λ_1 , and the only parameters appearing at order $1/m_b^3$ are ρ_D^3 and ρ_{LS}^3 . Expressions for non-truncated lepton spectrum have been used and higher moments have been included to gain sensitivity on $1/m_b^3$ parameters. A multi-parameter χ^2 fit has been performed to

the first three moments of the hadronic mass spectrum and lepton energy spectrum with the following results:

$$\begin{aligned} m_b(1\text{GeV}) &= (4.59 \pm 0.08 \pm 0.01) \text{ GeV} \\ m_c(1\text{GeV}) &= (1.13 \pm 0.13 \pm 0.03) \text{ GeV} \\ \mu_\pi^2(1\text{GeV}) &= (0.31 \pm 0.07 \pm 0.02) \text{ GeV}^2 \\ \rho_D^3(1\text{GeV}) &= (0.05 \pm 0.04 \pm 0.01) \text{ GeV}^3 \end{aligned}$$

A good consistency between all measurements has been obtained, demonstrating no need to introduce higher order terms to establish agreement with data, within the present accuracy. Fig. 7 shows the constraints extracted in the $\mu_\pi^2 - m_b$ and $\rho_D^3 - m_b$ planes.

The fit has been repeated also using the pole mass formalism [6] obtaining:

$$\begin{aligned} \bar{\Lambda} &= (0.40 \pm 0.10 \pm 0.02) \text{ GeV} \\ \lambda_1 &= (-0.15 \pm 0.07 \pm 0.03) \text{ GeV}^2 \\ \rho_1 &= (-0.01 \pm 0.03 \pm 0.01) \text{ GeV}^3 \\ \rho_2 &= (0.03 \pm 0.03 \pm 0.01) \text{ GeV}^3 \end{aligned}$$

compatible with CLEO results. The first uncertainties are from the fit, the second are systematic, coming from the variation of the residual parameters which have been fixed in the fit and from missing terms in the expansions.

The value of $|V_{cb}|$ obtained from the inclusive semileptonic decay width depends on the OPE parameters extracted above [14]. An approximate formula which displays the dependence on the different parameters is the following:

$$\begin{aligned} |V_{cb}| &= |V_{cb}|_0 \{1 + \\ &-0.65 [m_b(1) - 4.6 \text{ GeV}] + 0.40 [m_c(1) - 1.15 \text{ GeV}] \\ &+ 0.01 [\mu_\pi^2(1) - 0.4 \text{ GeV}^2] + 0.10 [\rho_D^3 - 0.12 \text{ GeV}^3] \\ &+ 0.06 [\mu_G^2(1) - 0.35 \text{ GeV}^2] + 0.01 [\rho_{LS}^3 + 0.17 \text{ GeV}^3]\} \end{aligned}$$

Using the world average value of the semileptonic width $\Gamma_{sl}^{exp} = (0.434 \pm 0.008) \times 10^{-10} \text{ MeV}$ the following value of $|V_{cb}|$ has been obtained:

$$|V_{cb}| = (41.6 \pm 0.4 \pm 0.6 \pm 0.4) \times 10^{-3}$$

where the first uncertainty is from the semileptonic width, the second is from the uncertainties in the non-perturbative parameters determined from the fit and the third is from the variation of α_s scale.

A similar analysis has been performed in [15] using CLEO and DELPHI measurements and consistent results have been obtained.

5 Conclusions

Measurements of moments of the photon energy spectrum in $B \rightarrow X_s \gamma$ decays, of the hadronic mass spectrum and of the lepton energy spectrum in $B \rightarrow X_c \ell \nu$

decays have been performed both at the $\Upsilon(4S)$ and at Z^0 center of mass energies. New results will also come from the B factories in the near future. Good agreement has been found between the values of the non-perturbative parameters of the heavy quark expansions derived from different measurements and no hint of a possible quark-hadron duality violation effect has emerged, within the present accuracy. As a consequence the uncertainties in the extraction of $|V_{cb}|$ from inclusive semileptonic with has been much reduced.

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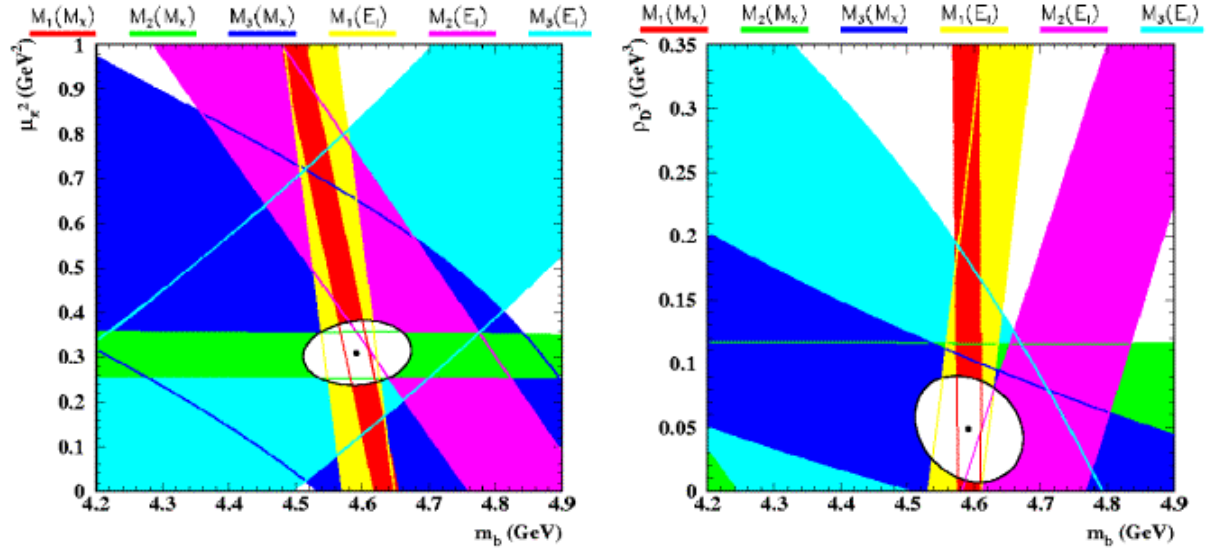


Figure 7. The constraints on the $\mu_\pi^2 - m_b$ (left) and $\rho_D^3 - m_b$ (right) plane obtained from the combination of the first three moments of the lepton energy spectrum and hadronic mass spectrum. The bands correspond to the total experimental accuracy on each moment, and are given by keeping all the other parameters at their central values. The ellipses represent the 1σ contour.